The Role of Social Networks on Regulation in the Telecommunication Industry: The Discriminatory Case.

by

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Abstract

In a previous work we studied the equilibrium behavior in a telecommunication market where two interconnected firms compete, using linear pricing schemes, in the presence of social networks among customers. We showed that social networks matter because equilibrium prices and welfare critically depend on how people are socially related. In this paper we extend the basic model to the nonlinear case, in particular, we consider the cases when firms can discriminate depending on the destiny of a call or, alternatively, when they can use two part tariffs. The standard regulated environment, in which the authority defines interconnection access charges as being equal to marginal costs and final prices are left to the market, is considered as a benchmark. The role of social networks is shown to be crucial in this new context too, despite the fact it has been usually ignored in the literature. Different regulatory interventions are evaluated in those environments.

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1 Introduction

Over the last years several articles have been focused on the study of the equilibrium interconnection strategies in telecommunication markets, in a framework where heterogeneity

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of consumers is recognized (see for example Dessein (2004) and Hahn (2004), among others).[1] This approach has been a significant improvement in the effort to obtain more realistic models.

In a recent article (Harrison et al. (2008)) it is shown that the position in a social network affects the amount of calls that a consumer optimally decide to make. This result is very intuitive because the more connections an individual has, the higher the number of calls she makes, ceteris paribus. Moreover, the number of calls to any particular member in the network should depend not only on prices, but also on how close they are in social terms. This simple fact generates some important implications in the analysis of equilibrium behavior when two interconnected firms compete for customers.

In the referred article we consider, as usual, that a firm A has two sources of revenues: its customer' payments and the access charges that a rival firm B pays to A in order to complete calls originated in B but terminated in A. Our benchmark case consisted of the standard regulated environment where interconnection access charges are defined by the authority as equal to marginal costs while final prices, constrained to linear pricing schemes, are left to the market. In such an environment we showed that social structure matters, because equilibrium prices, consumer surplus and producer surplus depend on network characteristics. In addition we studied the effectiveness of two alternative regulatory interventions when consumers are socially connected. The first was oriented to control access charges and the other was focused on reducing switching costs. Interestingly, our results showed that the later is much more effective even when standard regulation emphasize the former.

In this paper we generalize the horizontally differentiated competitive model of Harrison et al. (2008) by permitting the use of nonlinear pricing schemes. Two kinds of schemes are considered. First, when firms can price discriminate depending on the destiny of a call, and second, when two part tariffs are feasible. Although the theoretical model permits the presence of both effects together, the simulations becomes very complicated, so we numerically study the effect of both schemes separately.

Several papers are closely related to this article. The seminal papers are Laffont *et al.* (1998a,b) and Armstrong (1998). For an excellent review of the literature see Armstrong (2002). The equilibrium behaviour of interconnected firms in the presence of heterogeneous consumers has been analyzed by Dessein (2004) and Hahn (2004), among others. However, the use of social networks to model the connections among consumers has been introduced by Harrison et al. (2006, 2008).

The rest of the paper is organized as follows: In section 2 we develop the economic model, including the agent's demand, the firms' problem and the general game played by the two firms. In section 3 we specialize the model to study the equilibrium effects of

[1] For an excellent review of the literature see Armstrong (2002). The seminal papers are Laffont *et al.* (1998a,b) and Armstrong (1998).

discrimination depending on the destiny of the calls, while in section 4 we specialize the general model of section 2 to study the role of two part tariffs. Section 5 contains the simulations performed under both pricing schemes and the main equilibrium results. The conclusions are stated in section 6.

2 The Economic Model

The model closely follows Harrison *et al.* (2008) where we assumed the existence of a social network, represented by a graph g. Nodes in the graph represent agents (indexed by $i \in I$) and the links show how people are socially interconnected. There are two firms, A and B, offering horizontally differentiated communication services (for example two cellular companies) and consumers have to decide which firm to subscribe to. In order to make the affiliation decision, agents take into account the pricing schemes offered by each firm and his or her own preferences for the services provided. It is assumed that firms' pricing schemes are constrained to be linear and nondiscriminatory. On the other hand, the preferences are modeled in a similar way to a standard Hotelling horizontally differentiated model: each agent i in the social network (i.e. each node in g) is endowed with a realization of a taste random variable x_i, from a cumulative density function F with support in $[0, 1]$. In what follows we assume that firm A is "located" in 0 and firm B in 1. None of them provide the "ideal service" to agent i, positioned in x_i (this would be the case if some network were located precisely in x_i).

2.1 The Agent Demand

Consider the affiliation decision problem of agent i. If agent i decides to subscribe network $l = A, B$ then we will say that she belongs to the set $I_l \subseteq I$ of subscribers to l. Agent i's demand for calls is represented by the vector $q_i = (q_{ij})_{j \in I, j \neq i}$, where the generic element q_{ij} is the number of calls that agent i makes to agent j. Then the gross utility of agent i can be described as follows:[2]

$$U_i(q_i) = \sum_{j \in I, j \neq i} \delta^{t_{ij}} u(q_{ij}) \qquad \text{with} \qquad u(q_{ij}) = \frac{q_{ij}^{1-1/\eta}}{1 - 1/\eta} \tag{1}$$

where:

δ : represent a discount in utility when agent i calls other agents located farther in the network g. Accordingly, it satisfies $0 < \delta < 1$.

t_{ij} : it is the shortest distance (in terms of links) connecting agents i and j. We consider

[2] Note we are assuming in this formulation that all the individual in the network can receive calls even if he/she is not affiliated to A or B. This assumption is made for tractability but it is not so demanding if we consider that a prepaid phone can always receive calls in a calling party pays regime.

$t_{ij} = 0, 1, 2, ...$ so that if the agents are direct neighbors, the discount factor is $\delta^0 = 1$. On the other hand if the agents i and j are not connected then $t_{ij} = \infty$.
η : is a constant parameter representing the elasticity of demand, which is assumed to be greater than 1 and independent of j.

A typical pricing scheme applied for firm A (analogous for B) is given by $T(q_A, \widehat{q}_A) = F_A + p_A q_A + \widehat{p}_A \widehat{q}_A$ where F_A is a fixed charge and p_A is the price per call for a subscriber in network A when she is calling another subscriber in network A (on net call), while $\widehat{p}_A$ is the price per call for a subscriber in network A when she is calling a subscriber of B (off net call). The notation q_A and $\widehat{q}_A$ refers to the corresponding levels of on and off net calls, respectively.

For practical reasons, we are going to assume that a disconnected individual can still receive calls (for example in the fixed network). In such a case, the call is considered on net.[3]

Suppose that after observing the price schemes offered by the firms, agent i has to decide which firm to affiliate. In order to make that decision, she needs to figure out her net consumer surplus in both scenarios. If she decides to affiliate firm A, the vector of calls $q_i = (q_{ij})_{j \in I, j \neq i}$ to all her contacts in the network g is defined by:

$$W_i(p_A, \widehat{p}_A) = \max_{q_i} \left\{ U_i(q_i) - p_A \sum_{\substack{j \neq i \\ j \in I \setminus I_B}} q_{ij} - \widehat{p}_A \sum_{j \in I_B} q_{ij} \right\} \tag{2}$$

Solving this maximization problem, we obtain his/her demand's components:

$$q_{ij}(p) = \left(\frac{p}{\delta^{t_{ij}}} \right)^{-\eta} \quad \text{with} \quad p = p_A, \widehat{p}_A \tag{3}$$

Intuitively, for the same price p, agent i makes more calls to contacts located closer in the social network g than to those farther in it. Moreover, the possibility to discriminate depending on the destiny of a call makes the number of calls depending also on where agent j is affiliated. Therefore, plugging into equation 2 we get the indirect utility function:

$$W_i(p_A, \widehat{p}_A) = \sum_{\substack{j \neq i \\ j \in I \setminus I_B}} \delta^{\eta t_{ij}} \frac{p_A^{1-\eta}}{\eta - 1} + \sum_{j \in I_B} \delta^{\eta t_{ij}} \frac{\widehat{p}_A^{1-\eta}}{\eta - 1} \tag{4}$$

and an analogous result arise for firm B.

Consider the parameter t representing the unit cost associated to the fact that agent

[3] The assumption is not so demanding if we consider that standard plans typically consist of a number of minutes to be used on net or to fixed lines, and a different package for off net calls.

i, located in x_i, has to subscribe to network A located in 0 or network B located in 1. None of them provide the "ideal service" (this would be the case if some network were located precisely in x_i) so the cost of selecting a service different from i's preferred one is assumed to be $x_i t \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ if agent i selects network A or $(1-x_i)t \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ if network B is preferred.[4] It is important to note that in this model we assume that agent i incurs in a discounted disutility for calls due to the imperfect matching between her preferences and the service provided, where the discount appears because the imperfection is more annoying the closer is agent j to i in the social network. The total cost of imperfect matching is the sum of all the pairwise discounted costs. In addition, note that the cost to agent i of an imperfect service to call agent j is assumed independent of the number of calls.[5]

Let us define the net surplus for consumer i when affiliates to firm l (A or B) as:

$$w_i(p_l, \widehat{p}_l, F_l, x_i) \equiv W_i(p_l, \widehat{p}_l) - F_l - t x_i \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$$

The preference for A or B depends on whether x_i is to the right or to the left of a critical value x_i^* given by:

$$w_i(p_A, \widehat{p}_A, F_A, x_i^*) = w_i(p_B, \widehat{p}_B, F_B, 1 - x_i^*)$$

If $x_i < x_i^*$, agent i prefers network A even considering that network A does not provide him the ideal service (and has to pay $t x_i \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ by the imperfect matching). Solving for x_i^*, we got:

$$x_i^* = \frac{1}{2} + \sigma_i \left[W_i(p_A, \widehat{p}_A) - F_A - (W_i(p_B, \widehat{p}_B) - F_B) \right] \qquad \left(\text{with} \quad \sigma_i = \frac{1}{2t \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}} \right)$$

Let us define $\alpha_i = 0$ if agent i prefers network A and $\alpha_i = 1$ if agent i prefers network B.

Accordingly, the incentive compatibility constraint can be written as:

[4] This way to introduce transportation costs is different from Harrison ***et al.*** (2006), where the model consider a standard Hotelling transportation cost, but the utility function is normalized instead.

[5] Alternative approaches would be to make the transportation cost dependent on the utility obtained from the calls or dependent on the number of calls. Our selection is consistent with Laffont *et al.* (1998a). They do not consider, however, a discount factor because in their model agents are not connected through a social network.

$$\alpha_i = \begin{cases} 0 & \text{if} \quad x_i < x_i^* \\ 0 \text{ or } 1 & \text{if} \quad x_i = x_i^* \\ 1 & \text{if} \quad x_i > x_i^* \end{cases} \tag{5}$$

However, we also have to consider the option to remain disconnected. Agent i affiliates to telecommunication services (one of the two firms) if and only if

$$Max\left\{w_i(p_A, \widehat{p}_A, F_A, x_i), w_i(p_B, \widehat{p}_B, F_B, 1 - x_i)\right\} \geq 0$$

equivalently, we can define:

$$\Omega_i(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B, x_i, \alpha_i) = (1 - \alpha_i) w_i(p_A, \widehat{p}_A, F_A, x_i) + \alpha_i w_i(p_B, \widehat{p}_B, F_B, 1 - x_i)$$

and then, the individual rationality constraint for agent i is modeled by β_i such that:

$$\beta_i = \begin{cases} 0 & \text{if} \quad \Omega_i < 0 \\ 1 & \text{if} \quad \Omega_i \geq 0 \end{cases} \tag{6}$$

Accordingly, for example, in order that agent i affiliates firm A, it is necessary that she prefers A to B ($\alpha_i = 0$) and the net surplus from the affiliation to A should be no negative ($\beta_i = 1$).

2.2 The Firm's Problem

Assuming that each firm pursues maximization of profits, when access charges are given by a_A and a_B, firm A (resp. B) will select its prices $p_A, \widehat{p}_A, F_A$ (resp. $p_B, \widehat{p}_B, F_B$) such that:[6]

$$\begin{aligned} &\max_{p_A, \widehat{p}_A, F_A \geq 0} \pi_A(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B, a_A, a_B) = \\ &\textstyle\sum_{i \in I_A} \left\{ \sum_{\substack{j \in I \setminus I_B \\ j \neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + \sum_{j \in I_B} q_{ij}(\widehat{p}_A)(\widehat{p}_A - c_A^o - a_B) + F_A - f \right\} + \\ &\textstyle\sum_{i \in I_B} \sum_{j \in I_A} q_{ij}(\widehat{p}_B)(a_A - c_A^f) \end{aligned} \tag{7}$$

where:[7]

[6] Note that we have assumed that individuals not affiliated to firms can be called at marginal termination costs.

[7] In what follows, when we solve an optimization problem, we always assume that g, f, c_A^o, c_B^o, c_A^f, c_B^f, $\{x_i\}_{i=1}^I$ and t are all given exogenously.

f : is the fixed cost incurred by a firm when it affiliates a new subscriber.
c^o_A : is the cost of originating a call for firm A (c^o_B is defined analogously).
c^f_A : is the cost of terminating or finishing a call for firm A (c^f_B is defined analogously).
a_A : is the price or access charge that firm A charges firm B in order to terminate a call from a subscriber of B to a subscriber of A (a_B is defined analogously).

The structure given in problem (7) is not convenient, because the individual rationality and incentive compatibility constraints for customers are embeded in the sets where the sums are calculated. In what follows we want to make explicit those constraints to facilitate the algorithm to find the Nash equilibrium prices in the competition between firms. In order to do that, our goal will be to express both constraints in linear form, so as to write firm A's problem as:

$$\underset{p_A,\widehat{p}_A,F_A\geq 0}{Max} \; \pi_A(p_A,\widehat{p}_A,p_B,\widehat{p}_B,F_A,F_B,a_A,a_B;\alpha,\beta) \tag{8}$$

$$\begin{array}{lllll} s.t. & H_1\alpha & \leq & z_1, & \alpha\in\{0,1\}^I \qquad (IC \text{ constraints}) \\ & H_2\beta & \leq & z_2, & \beta\in\{0,1\}^I \qquad (IR \text{ constraints}) \end{array}$$

With this goal in mind, we separate the problem in two parts. First, we write the affiliation decision as a system of inequality constraints and then, we write the objective function as in (8). Even so, the dimension of the problem makes it unsolvable in the general case, so we focus on two particular kinds of discrimination. In the first, we study the role of price discrimination depending on the destiny of a call, but we set F_A and F_B as equals to zero. In the second, we study the role of two part tariffs, but we impose $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$.

3 Case I: Discriminating in Destiny

In order to simplify this case we can assume that for all the consumers the individual rationality constraint is not binding, i.e., $\beta_i = 1\ \forall i \in I$. Otherwise, adding a constant to the utility function is enough to satisfy this condition. It is easy to see that α_i now represents affiliation decisions and we can write:

$$\alpha_i = \begin{cases} 0 & \text{if} \quad b_i < L_i^t\alpha_{-i} \\ 0 \text{ or } 1 & \text{if} \quad b_i = L_i^t\alpha_{-i} \\ 1 & \text{if} \quad b_i > L_i^t\alpha_{-i} \end{cases} \tag{9}$$

where α_{-i} is a $I-1$ column vector containing the affiliation decisions of agents other than i, L_i is a $I-1$ column vector and $b_i \in \mathbb{R}$ with:

$$\begin{aligned} b_i &= x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, \widehat{p}_B) \\ L_i &= e_{-i}(\widehat{p}_A, p_B) - e_{-i}(p_A, \widehat{p}_B) \end{aligned}$$

where:

$$e_{-i}(p,q) = \begin{pmatrix} e_{i,1}(p,q) \\ \vdots \\ e_{i,i-1}(p,q) \\ e_{i,i+1}(p,q) \\ \vdots \\ e_{i,I}(p,q) \end{pmatrix}_{I-1} \qquad \alpha_{-i} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{i-1} \\ \alpha_{i+1} \\ \vdots \\ \alpha_I \end{pmatrix}_{I-1} \qquad \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{I-1}$$

$$e_{i,j}(p,q) = \frac{\sigma_i \delta^{\eta t_{ij}}}{\eta - 1} \left(p^{1-\eta} - q^{1-\eta} \right)$$

The constraint (9) is still hard to incorporate in an optimization program. We would like to have a linearized version of this constraint which should be impossed $\forall i \in I$.

Consider M sufficiently high[8] such that, for given i, the expression (9) is equivalent to the following couple of inequations:

$$\begin{aligned} L_i^t \alpha_{-i} &\geq b_i - M\alpha_i \\ L_i^t \alpha_{-i} &\leq b_i + M(1-\alpha_i) \end{aligned} \tag{10}$$

In effect, when $b_i < L_i^t \alpha_{-i}$ holds, agent i is forced to choose $\alpha_i = 0$ otherwise (i.e. by selecting $\alpha_i = 1$) the second inequality in (10) is violated. An analogous argument applies when $b_i > L_i^t \alpha_{-i}$. In the case when $b_i = L_i^t \alpha_{-i}$ the inequalities in (10) are satisfied with either $\alpha_i = 0$ or $\alpha_i = 1$.

As a result, the vector of affiliation decisions must satisfies the following system of linear inequations:

[8] A feasible definition of M is given in the Appendix.

$$H_1^I \alpha \leq z_1^I$$

where:

$$H_1^I = \begin{bmatrix} -M & -(L_1^D)^t & \\ M & (L_1^D)^t & \\ -(L_2^U)^t & -M & -(L_2^D)^t \\ (L_2^U)^t & M & (L_2^D)^t \\ \vdots & \vdots & \vdots \\ -(L_i^U)^t & -M & -(L_i^D)^t \\ (L_i^U)^t & M & (L_i^D)^t \\ \vdots & \vdots & \vdots \\ & -(L_I^U)^t & -M \\ & (L_I^U)^t & M \end{bmatrix}_{2I \times I} \qquad z_1^I = \begin{bmatrix} -b_1 \\ b_1 + M \\ -b_2 \\ b_2 + M \\ \vdots \\ \vdots \\ -b_I \\ b_I + M \end{bmatrix}_{2I \times 1} \qquad \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_I \end{pmatrix}_{I \times 1}$$

and:

$$L_i = \begin{pmatrix} L_{i,1} \\ \vdots \\ L_{i,i-1} \\ L_{i,i+1} \\ \vdots \\ L_{i,I} \end{pmatrix} \equiv \begin{pmatrix} L_i^U \\ L_i^D \end{pmatrix} \qquad \alpha_{-i} \equiv \begin{pmatrix} \alpha_{-i}^U \\ \alpha_{-i}^D \end{pmatrix} \qquad \alpha = \begin{pmatrix} \alpha_{-i}^U \\ \alpha_i \\ \alpha_{-i}^D \end{pmatrix}$$

It is convenient to emphasize that H_1^I depends on the vector of prices $(p_A, p_B, \widehat{p}_A, \widehat{p}_B)$ because L_i does for each i. Analogously, z_1^I also depends on the vector of prices because b_i does for each i.

Consider now the objective function for firm A, established in equation (7), but in the absense of fixed charges:

$$\pi_A^I(p_A,p_B,\widehat{p}_A,\widehat{p}_B) = (p_A - c_A^o - c_A^f)p_A^{-\eta}\sum_{i\in I_A}\sum_{\substack{j\neq i\\ j\in I_A}}\delta^{\eta t_{ij}} + (\widehat{p}_A - c_A^o - a_B)\widehat{p}_A^{-\eta}\sum_{i\in I_A}\sum_{j\in I_B}\delta^{\eta t_{ij}}$$
$$-\sum_{i\in I_A} f + (a_A - c_A^f)\widehat{p}_B^{-\eta}\sum_{i\in I_B}\sum_{j\in I_A}\delta^{\eta t_{ij}}$$

and using the definition of α_i we have:

$$\begin{aligned}\pi_A^I(p_A,p_B,\widehat{p}_A,\widehat{p}_B;\alpha) &= (p_A - c_A^o - c_A^f)p_A^{-\eta}\sum_{i\in I}\sum_{\substack{j\neq i\\ j\in I}}\delta^{\eta t_{ij}}(1-\alpha_i)(1-\alpha_j)\\ &+(\widehat{p}_A - c_A^o - a_B)\widehat{p}_A^{-\eta}\sum_{i\in I}\sum_{j\in I}\delta^{\eta t_{ij}}(1-\alpha_i)\alpha_j\\ &-\sum_{i\in I}(1-\alpha_i)f + (a_A - c_A^f)\widehat{p}_B^{-\eta}\sum_{i\in I}\sum_{j\in I}\delta^{\eta t_{ij}}\alpha_i(1-\alpha_j)\end{aligned}$$

Accordingly, the problem for firm A has been transformed into:

$$\underset{p_A,\widehat{p}_A\geq 0}{Max}\ \pi_A^I(p_A,p_B,\widehat{p}_A,\widehat{p}_B,a_A,a_B;\alpha) \tag{11}$$

$$s.t.\quad H_1^I\alpha \leq z_1^I, \quad \alpha\in\{0,1\}^I \qquad (IC \text{ constraints})$$

4 Case II: Two part Tariffs

It is easy to see that the incentive compatibility constraint is a particular case of the analysis in the previous subsection with $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$. In such a case $L_i = 0$ $\forall i \in I$ so matrix H_1^{II} is considerably simpler. Additionaly the expression for b_i would naturally become $b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A,p_B) - \sigma_i(F_B - F_A)$. The interpretation for α_i, however, is back to one of preferences instead of affiliation decisions.

An analogous procedure let us to establish an N sufficiently high such that equation (6) is equivalent to:

$$\begin{aligned} 0 &\geq \Omega_i - N\beta_i \\ 0 &\leq \Omega_i - N(1-\beta_i)\end{aligned} \tag{12}$$

As a result, the vector of market participation decisions must satisfy the following system of linear inequations:

$$H_2^{II}\beta \leq z_2^{II}$$

where:

$$H_2^{II} = \begin{bmatrix} -N & & & \\ N & & & \\ & -N & & \\ & N & & \\ \vdots & \vdots & \vdots & \\ & & -N & \\ & & N & \end{bmatrix}_{2I\times I} \qquad z_2^{II} = \begin{bmatrix} -\Omega_1 \\ \Omega_1 + N \\ -\Omega_2 \\ \Omega_2 + N \\ \vdots \\ -\Omega_I \\ \Omega_I + N \end{bmatrix}_{2I\times 1} \qquad \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_I \end{pmatrix}_{I\times 1}$$

In this case, it is convenient to emphasize that H_2^{II} is independent of a particular vector of prices (p_A, p_B). However, z_2^{II} depends now on the vector of prices, fixed charges and also on α_i because Ω_i does for each i.

Consider now the objective function for firm A, established in equation (7), but in the absense of destiny based discrimination:

$$\pi_A^{II}(p_A, p_B, F_A, F_B, a_A, a_B) =$$
$$\sum_{i\in I_A}\left\{\sum_{\substack{j\in I\setminus I_B\\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + \sum_{j\in I_B} q_{ij}(p_A)(p_A - c_A^o - a_B) + F_A - f\right\} +$$
$$\sum_{i\in I_B}\sum_{j\in I_A} q_{ij}(p_B)(a_A - c_A^f)$$

Assuming symmetric access charges it becomes:

$$\pi_A^{II}(p_A, p_B, F_A, F_B, a_A, a_B) =$$
$$\sum_{i\in I_A}\left\{\sum_{\substack{j\in I\\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + F_A - f\right\} +$$
$$\sum_{i\in I_A}\sum_{j\in I_B}\left(q_{ji}(p_B) - q_{ij}(p_A)\right)(a - c_A^f)$$

and using the definitions of α_i and β_i we have:

$$\pi_A^{II}(p_A, p_B, F_A, F_B, a_A, a_B; \alpha, \beta) =$$
$$\sum_{i\in I}(1-\alpha_i)\beta_i \left\{ \sum_{\substack{j\in I \\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + F_A - f \right\} +$$
$$\sum_{i\in I}(1-\alpha_i)\beta_i \sum_{\substack{j\in I \\ j\neq i}} \alpha_j\beta_j \left(q_{ji}(p_B) - q_{ij}(p_A)\right)(a - c_A^f)$$

Accordingly, the problem for firm A has been transformed into:

$$\underset{p_A, F_A \geq 0}{Max}\ \pi_A^{II}(p_A, p_B, F_A, F_B, a_A, a_B; \alpha, \beta) \tag{13}$$

$$\begin{array}{llll} s.t. & H_1^{II}\alpha \leq z_1^{II}, & \alpha \in \{0,1\}^I & (IC \text{ constraints}) \\ & H_2^{II}\beta \leq z_2^{II}, & \beta \in \{0,1\}^I & (IR \text{ constraints}) \end{array}$$

5 Simulation Results

In this section we report the main simulation results. It should be noted that problems (11) and (13) are nonlinear not only in the objective function, but also in the constraints, because they depend on prices. For any given vector of prices the constraint can be solved in α and/or β. Once α and/or β has been selected, we can evaluate the goal function for the corresponding vector of prices, access charges, α and β. We look for symmetric equilibrium strategies among firms.

The default values for the parameters are given in Table 1. In the subsequent analysis below, we depart from this setting in some key variables.

Table 1: Default Values for Parameters

elasticity of demand	$-\eta = -1.2$
discount factor	$\delta = 0.9$
connectivity degree	$d = 8$
origination cost	$c_A^o = c_B^o = 0.75$
termination cost	$c_A^f = c_B^f = 0.75$
fix cost	$f = 50$
access charges	$a_A = a_B = 0.75$
number of individuals	$I = 100$
transportation cost	$t = \begin{cases} 0.5 \text{ in case } I \\ 0.25 \text{ in case } II \end{cases}$

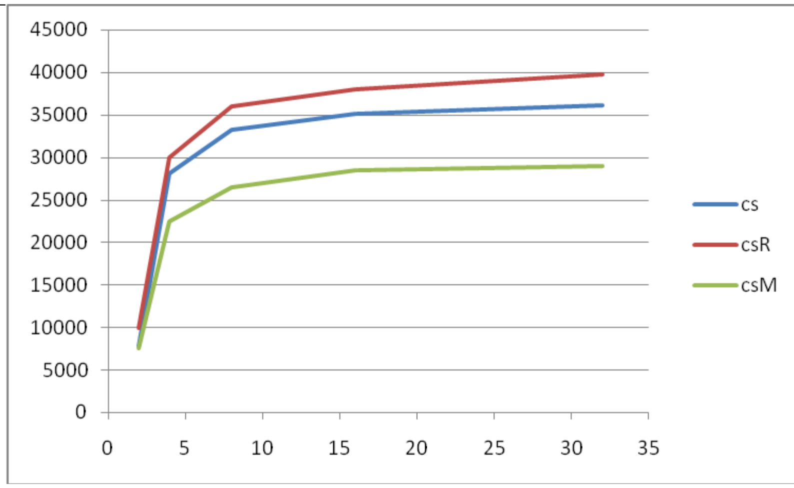

Figure 1: The impact of Connectivity degree on Consumer Surplus (CS).

All the numbers in Table 1 were selected trying to conform a reasonable setting. For example, Ingraham and Sidak (2004) have estimated that the elasticity of demand in US for wireless services is between -1.12 and -1.29. The fixed cost (f) has been selected in order to represent 10% of ARPU (Average Revenue per User). On the other hand, origination, termination and transportation costs are in the same order of magnitude reported by De Bijl and Peitz (2002) in their simulations.

5.1 Results for Case I: Discrimination in Destiny

In this case the structure of matrix H_1^I is complex enough for the constraint to admit multiple equilibria. The criteria here was to select α so as to minimize $\sum_{i=1}^{I} \alpha_i$. In other words, the most favorable selection fo firm A in terms of market share.

Main results are summarized in figures 1 to 3. In order to be able to compare with reference cases, we also provide the results for the Ramsey and the monopoly case. In Figure 1 we show the dependance of consumer surplus on the connectivity degree (d). The connectivity degree is the average number of agents that an individual directly connect in the social network. It is clear from the figure that social structure matters.

Figure 2 reports the equilibrium results for both average prices (p and $\widehat{p}$) when access charges are permitted to change. Both of them are above the Ramsey prices and by far below monopoly prices, but the most interesting finding is that for sufficiently low access charges it is cheaper to call off net than on net. The reason is simple, receiving calls from the rival firm is expensive, because the termination cost is higher than the access charge.

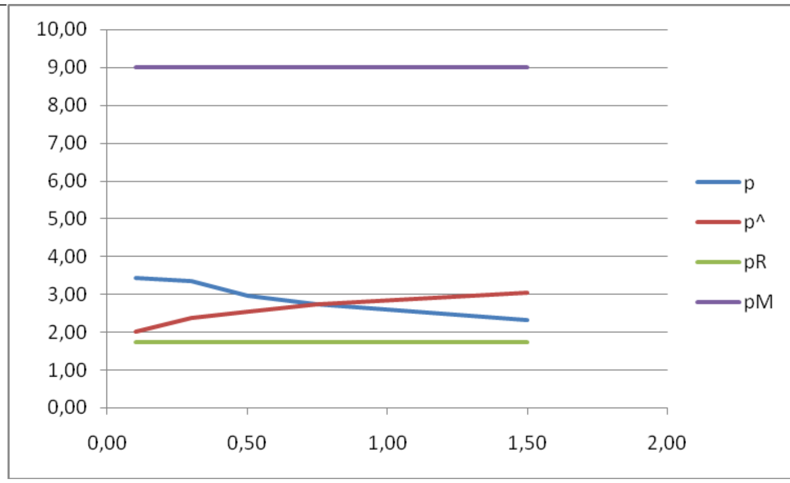

Figure 2: Average equilibrium prices (p and $\widehat{p}$) as a function of access charges.

As a result, firms try to attract high demand customers to avoid a high flow coming from the rival network.

Figure 3 reports the equilibrium results for both average prices (p and $\widehat{p}$) when switching (transportation) costs are permitted to change. Both of them are above the Ramsey prices, but they are closer than in Figure 2. It is interesting that for low switching costs it is cheaper to call off net than on net. The reason for this behavior, however, is quit different from the case of access charge regulation described in Figure 2. If in equilibrium $\widehat{p}$ were higher than p then all the consumers would affiliate one network and, in the static framework under analysis, the competition would be intensified because who lose the battle is out of the market. Moreover, given the rule to select among multiple equilibria, the surviving firm would be A.

5.2 Results for Case II: Two part Tariffs

The case of nonlinear pricing schemes is different in several aspects. First, in this case an important issue is participation at all in the market ($\beta_i = 1$ versus 0). Second, uniqueness in the solution for affiliation decisions is guaranteed. This is because both matrix H_1^{II} and H_2^{II} are "diagonal". The net effect is that new algorithms are simpler than those developed for case I.

Some very preliminar results are summarized in figures 4 to 6. In Figure 4 we show the dependence of consumer surplus on the connectivity degree (d). As in the previous case, it is clear from the figure that social structure matters.

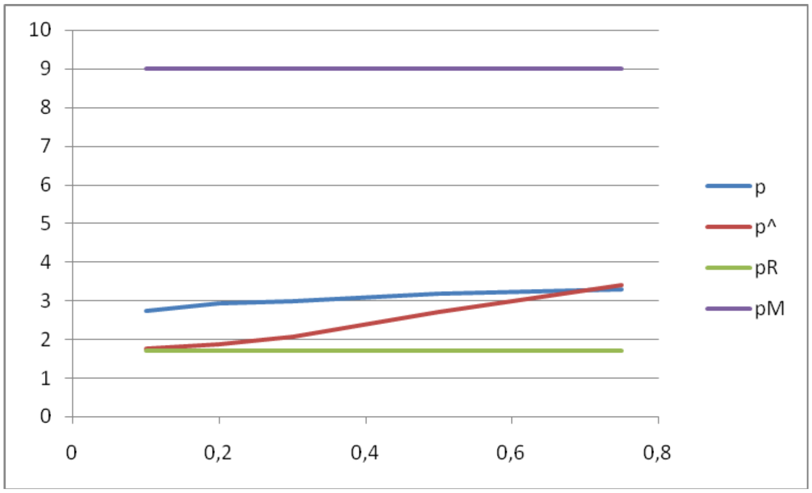

Figure 3: Average equilibrium prices (p and $\widehat{p}$) as a function of transportation costs.

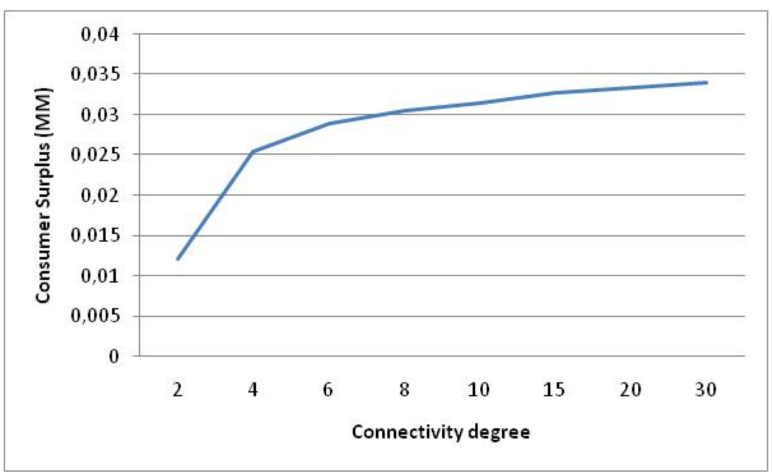

Figure 4: The impact of connectivity degree on Consumer Surplus (CS).

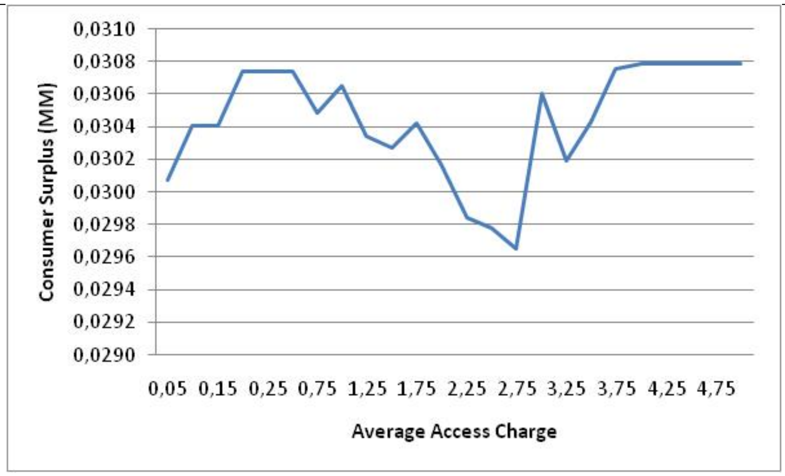

Figure 5: Consumer Surplus as a Function of Average Access Charge.

In order to study the relative efficiency of the two kinds of regulations: access charges v/s transportation costs, it is convenient to focus the analysis on the effect of each variable on Consumer Surplus. Figures 5 and 6 show that reductions in transportation costs have a higher and predictable effect on Consumer Surplus.

6 Conclusion

In this paper we have studied the equilibrium behaviour of agents in a market characterized by the competition between two interconnected firms providing services to consumers related through a social network. Differently from Harrison et al. (2008), we consider the case where firms can use nonlinear pricing schemes. As in the referred article, the results showed that equilibrium outcome depends on the connectivity parameter d, showing that social networks matter in the way how the market performs. As in the previous paper too, our results showed that access charges regulation is less effective than a policy oriented to reduce switching costs.

Some new results are particularly interesting. For example, when discrimination in destiny is permitted, and access charge regulation has set access charges below marginal costs, then in equilibrium off net calls becomes cheaper than on net calls. On the other hand, when two part tariffs are feasible, consumer surplus is more effectively increased by a policy oriented to reduce switching costs rather than a policy focused on reducing access charges.

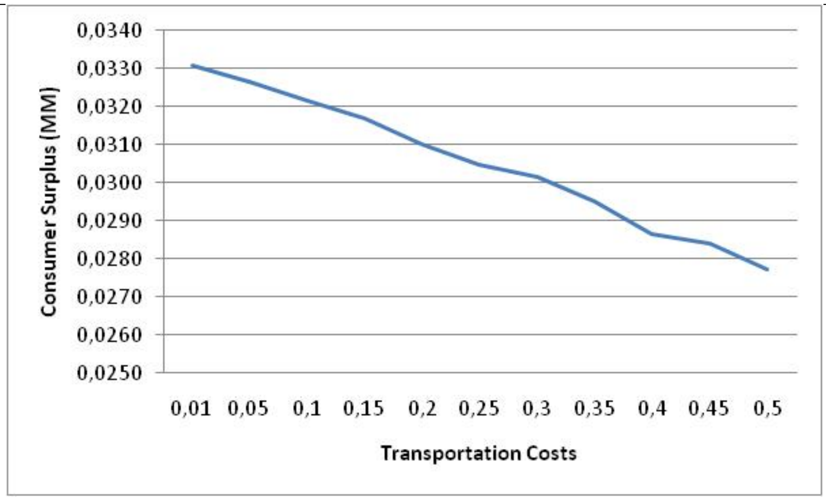

Figure 6: Consumer Surplus as a Function of Transportation Costs.

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8 Appendix 1

The goal of this section is to define valid values for the bounds M and N introduced in equations (10) and (12) respectively.

In the case of M the bounding process is quit similar to the case with linear pricing schemes. It is easy to show that a valid bound is:

$$M = \frac{1}{2} + \frac{\sigma}{\eta - 1}(I-1)\left[\underline{P}^{1-\eta} - \overline{P}^{1-\eta}\right] + \sigma(\overline{F} - \underline{F})$$

where $\sigma = 1/2t$ and the underbar and upperbar represents the minimum and maximum possible value for the corresponding variable.

On the other hand, from equation (12) and the definition of Ω_i we can write:

$$
\begin{aligned}
|\Omega_i(p_A, F_A, p_B, F_B, \alpha_i)| &= |(1-\alpha_i)w_i(p_A, F_A) + \alpha_i w_i(p_B, F_B)| \\
&< |w_i(p_A, F_A)| + |w_i(p_B, F_B)| \\
&\leq V_i(p_A) + F_A + tx_i \sum_{\substack{j \in I \\ j \neq i}} \delta^{t_{ij}} + V_i(p_B) + F_B + tx_i \sum_{\substack{j \in I \\ j \neq i}} \delta^{t_{ij}} \\
&\leq I \frac{\left[\overline{P}_A^{1-\eta} + \overline{P}_B^{1-\eta}\right]}{\eta - 1} + 2\left[\overline{F} + tI\right] \\
&\leq +2\left[I \frac{\overline{P}^{1-\eta}}{\eta - 1} + \overline{F} + tI\right]
\end{aligned}
$$

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